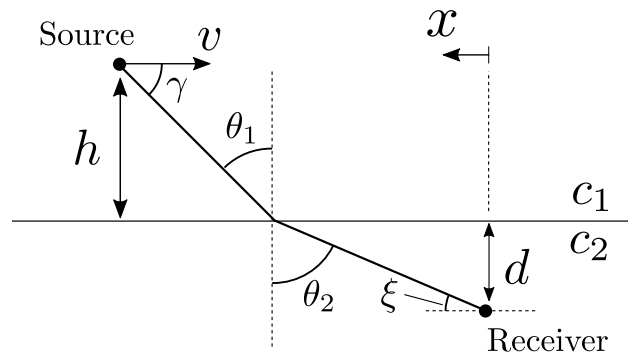


# International Student Challenge Problem in Acoustic Signal Processing 2019

Scott Schoen Jr

Mechanical Engineering, Georgia Tech  
Atlanta, GA USA | scottschoenjrgatech.edu

Problem text has been adapted from the problem publication<sup>1</sup> and Ref. 2. MATLAB source code for reproduction of the figures is provided in Appendix B. The geometry and coordinates used for this problem are shown in Fig. 1. The choice of the  $x$ -direction is arbitrary, but taken leftward such that the plane's position is positive during its approach.



**Figure 1:** Geometry of the problem. The plane sound source at height  $h$  travels with horizontal velocity  $v$  in the negative  $x$ -direction, and its acoustic signature is measured by a receiver at depth  $d$ . After Refs. 1 and 3.

## Task 1

Given that a turboprop aircraft is in level flight at a speed of 239 knots (123 m/s) and an altitude of 496 ft (151 m); that the depth of the hydrophone is 20 m below the (flat) sea surface; that the isospeed of sound propagation in air is 340 m/s; and that in seawater, it is 1520 m/s, predict the variation with time of the instantaneous frequency using Urlick's two isospeed sound propagation media approach and comment on its goodness of fit to the measurements.

The ray that leaves the plane and reaches the hydrophone will be refracted according to Snell's law,<sup>†</sup>

$$\frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2}, \quad (1.1)$$

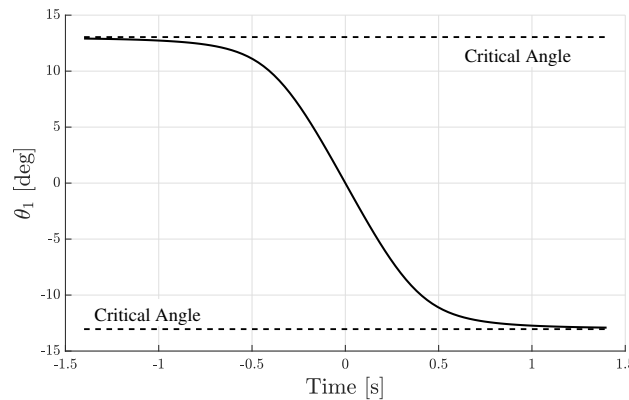
where 1 subscripts indicate values for the air half-space, and 2 for water. Align the  $x$ -axis with the direction of the plane travel and its origin directly above the hydrophone (see Fig. 1),

<sup>†</sup>As seems to be the case with many important results, the choice of eponym for Eq. (1.1) is not totally untested. Evidently Abu Said al-Ala Ibn Sahl (984), Thomas Harriot (ca. 1602), and René Descartes (1637) all have claims towards its modern formulation.<sup>4</sup>

such that the ray that leaves the plane and arrives at the sensor must obey

$$x(t) = h \tan \theta_1 + d \tan \theta_2. \quad (1.2)$$

With our choice of coordinates, we have  $x(t) = -vt$ . I'm not sure if there's a nice closed form expression to be found from Eqs. (1.1) and (1.2), but since we now have two equations, we can use a root finding algorithm<sup>†</sup> to find  $\theta_1$  as a function of time. We should also note that since  $c_2 > c_1$ , there is a critical angle around  $13^\circ$ ; beyond this angle, we should expect no sound from the plane to reach the receiver.<sup>‡</sup> The so calculated angle of incidence  $\theta_1$  as a function of time is shown in Fig. 1.1.



**Figure 1.1:** Computed angle of incidence  $\theta_1$  of the direct path ray as a function of time. The critical angles are indicated by the dashed lines.

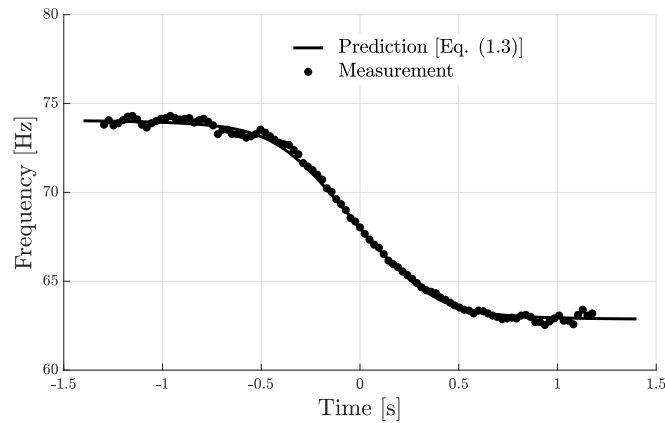
Now, with  $v$  as the velocity of the source, and  $\hat{r}$  as the unit vector along the incident ray, its apparent frequency will be Doppler shifted according to<sup>6</sup>

$$\omega = \frac{\omega_0}{1 - \mathbf{v} \cdot \hat{\mathbf{r}}/c_1} = \frac{\omega_0}{1 - (v/c_1) \sin \theta_1}. \quad (1.3)$$

The predicted frequency from Eq. (1.3) shows excellent agreement with the measurements, with an RMS error of only 0.24 Hz over the interval; see Fig. 1.2. The provided measurements end around  $\pm 1.4$  s, times which are consistent with the critical angles seen in Fig. 1.1. The variability of the measured frequency at the extreme times ( $|t| \gtrsim 0.5$  s) from the prediction may be due to surface variability, since water waves occur on meter scales—similar to that of the incident wavelength.

<sup>†</sup>I've just taken advantage of the exceptionally convenient MATLAB built-in function `fzero`; see Appendix B. This algorithm uses a combination of several canonical techniques and is adapted from Ref. 5.

<sup>‡</sup>That is, a single ray from the point source will not reach the receiver. Though the plane has finite extent, a rough estimation of the propeller dimension  $a$  gives  $ka \sim 1$ . Thus while the source is likely not omnidirectional, a point source model (valid for  $ka \ll 1$ ) is not egregious. Additionally, arrivals from surface scattering and the bottom bounce path (which are not considered here) may be present in the measured data.



**Figure 1.2:** Comparison of the predicted instantaneous frequency [Eq. (1.3), solid line], with the the observed frequencies from the file (circles) as a function of time.

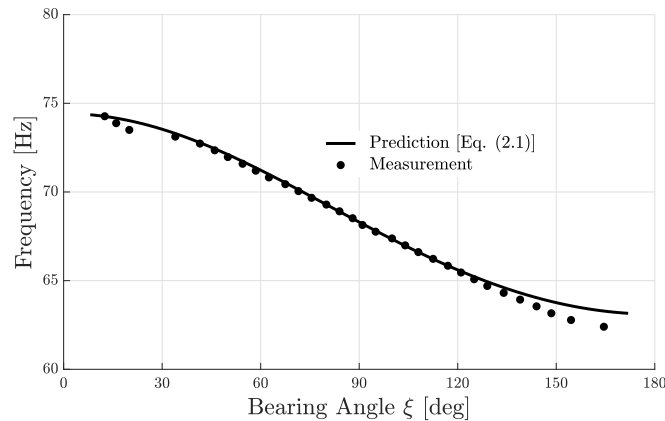
## Task 2

Predict the variation with the elevation angle of the instantaneous frequency of the source signal using Urick's two isospeed media approach and comment on its goodness of fit to the actual data measurements.

The ray arriving at the array from grazing angle  $\xi = \frac{\pi}{2} - \theta_2$  has an depression angle from the source  $\gamma$  (see Fig. 1). Computing  $\theta_1$  as in Problem 1, we have (with  $\gamma = \frac{\pi}{2} - \theta_1$ ) the shifted frequency

$$\omega = \frac{\omega_0}{1 - (v/c_1) \cos \gamma}. \quad (2.1)$$

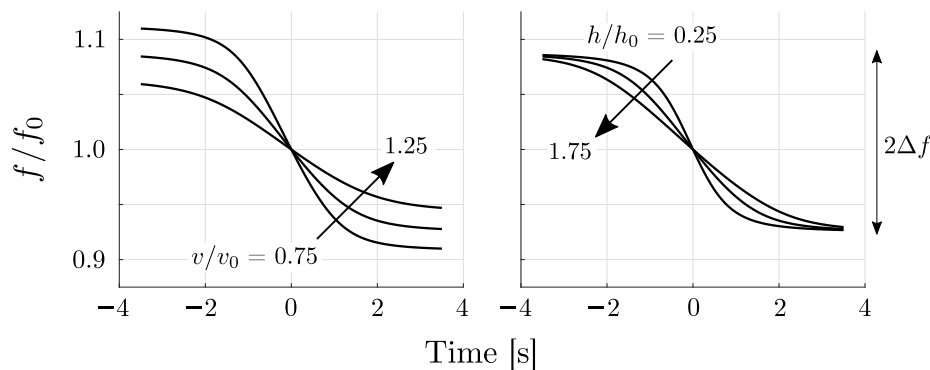
Then since  $\cos \gamma = (c_1/c_2) \cos \xi$  we can evaluate the expected source frequency as a function of the array bearing angle  $\xi$ . Plotting Eq. (2.1) (solid line in Fig. 2.1) again indicates very good agreement with measured data (dots) with RMS error of 0.28 Hz over all angles.



**Figure 2.1:** Comparison of the model prediction [Eq. (2.1), solid line] and measurement (dots) as a function of the bearing angle  $\xi$ ; see Fig. 1.

### Task 3

Given that the hydrophone is at a depth of  $d = 90$  m, estimate the speed of the aircraft (in meters per second), the altitude of the aircraft (in meters), the source (or rest) frequency (in hertz), and the time (in seconds) at which the aircraft is at its closest point of approach to the hydrophone (i.e., when the source is directly above the sensor).

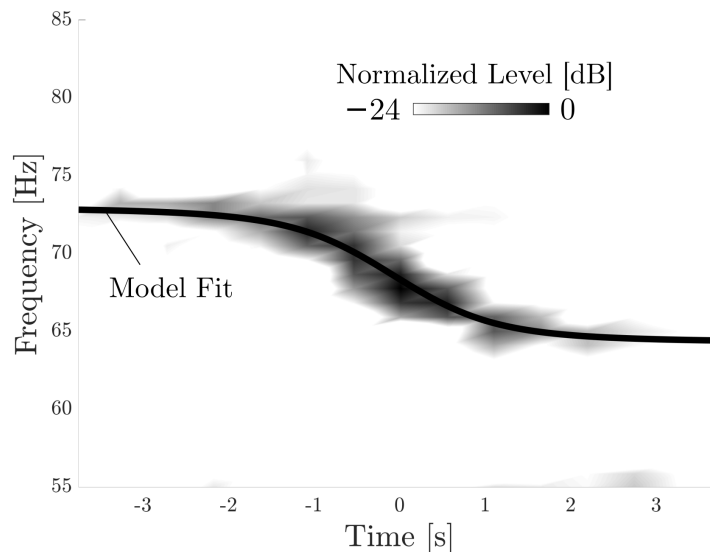


**Figure 3.1:** (Left) Predicted instantaneous frequency for sources with velocities of  $0.75v_0$ ,  $v_0$ , and  $1.25v_0$ , with  $v_0 = 123$  m/s. (Right) Predicted instantaneous frequency for sources with heights of  $0.25h_0$ ,  $h_0$ , and  $1.75h_0$ , with  $h_0 = 500$  m.

A full nonlinear fitting of Eqs. (1.1) to (1.3) to the measured data would give optimal estimation of these parameters. However, a simpler approach is to check the influence of each variable and see if it may be estimated independently from the others. Figure 3.1 demonstrates the effect of the source speed  $v$  and altitude  $h$  on the predicted instantaneous frequency. From these it is seen that the magnitude of the frequency shift is governed by plane's

speed (Fig. 3.1, left), while the time interval over which this change occurs is determined by its altitude (Fig. 3.1, right). Then, since the shift is symmetric about the approach, the source frequency  $f_0$  can be recovered by averaging the maximum and minimum observed frequencies.

Inspecting the frequency shift in the measured data (Fig. 3.2) indicates a frequency shift of  $\Delta f = \pm 4.12$  Hz about a center frequency of  $f_0 = 69.12$  Hz. This center frequency is achieved at  $t = 54.6$  s in the file—i.e., the time at which the source flies directly overhead. Fitting the velocity to achieve this shift gives a best fit velocity of  $v = 95.97$  m/s. A value of  $h = 150$  m was used for the fit; however, the acoustic signature is not a strong function of altitude (cf. Fig. 3.1, right), thus the source altitude uncertainty is on the order of 100 m.

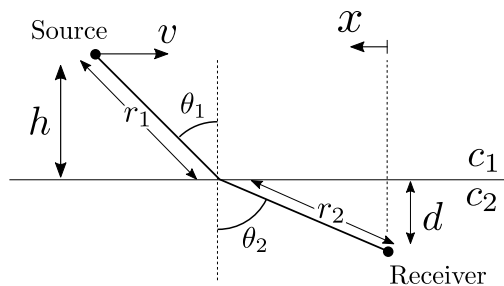


**Figure 3.2:** Comparison of instantaneous frequency calculated with Eq. (1.3) (black line) overlaid with normalized hydrophone time series data (grayscale). For the model, the found best fit parameters were  $f_0 = 69.12$  Hz,  $v = 95.97$  m/s, and  $h = 150$  m. The time origin was set to  $t_0 = 54.6$  s. The dynamic range for the colorplot is 24 dB.

## A Alternate Derivation of Frequency Shift

Call the path lengths of the ray in the air and water  $r_1$  and  $r_2$ , respectively as in Fig. A.1. The excess phase of the ray that arrives at the receiver will be

$$\begin{aligned} \phi &= k_1 r_1 + k_2 r_2 \\ &= \frac{\omega_0}{c_1} \frac{h}{\cos \theta_1} + \frac{\omega_0}{c_2} \frac{d}{\cos \theta_2}. \end{aligned} \quad (\text{A.1})$$

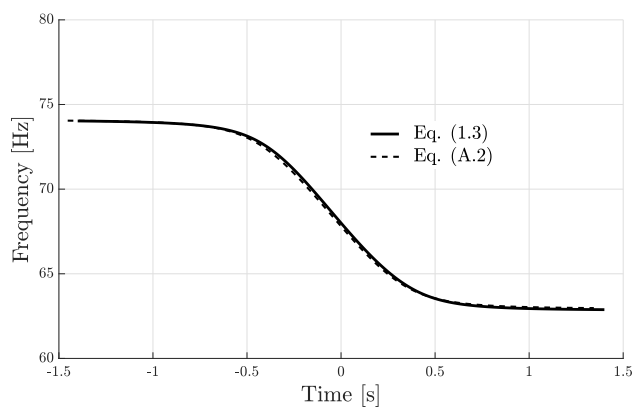


**Figure A.1:** Geometry for the ray length consideration of the frequency shift.

Thus the instantaneous shift in frequency is given by

$$\omega(t) = \frac{d\phi}{dt} = \frac{d}{dt} \left[ \frac{\omega_0}{c_1} \frac{h}{\cos \theta_1} + \frac{\omega_0}{c_2} \frac{d}{\cos \theta_2} \right]. \quad (\text{A.2})$$

Because numerical solutions for  $\theta_1(t)$  and  $\theta_2(t)$  can be obtained from Eqs. (1.1) and (1.2), we can forgo the algebra and simply approximate Eq. (A.2) via  $\omega \approx \Delta\phi/\Delta t$ . Figure A.2 shows agreement between Eq. (A.2) and the result using the Doppler shifted ray [Eq. (1.3)].



**Figure A.2:** Comparison of computed instantaneous frequency with Eq. (1.3) (solid line) and Eq. (A.2) (dashed line).

## B MATLAB Code

Code is available for download on GitHub:  
<https://github.com/scottschoenjr/asaChallenge2019>.

### B.1 Problem 1

```
%*****  
%  
% ASA Student Challenge Problem 2019  
%  
% Problem 1 – Plots the incident angle of the arrival ray as a function  
% of time, as well as the resulting Doppler shifted frequency as a  
% function of time, for comparison with the provided measurements.  
%  
%  
% Scott Schoen Jr | Georgia Tech | 30 September 2019  
%  
%*****  
  
clear all  
close all  
clc  
  
dataFile = '../media/timeVsFreq.txt';  
  
% Load in data  
allData = importdata( dataFile );  
tMeas = allData( :, 1 );  
fMeas = allData(:, 2 );  
  
% Problem parameters  
d = 20; % [m]  
h = 151; % [m]  
v = 123; % [m/s]  
f0 = 68; % [Hz]  
  
c1 = 343; % [m/s]  
c2 = 1520; % [m/s]  
  
% Define plane position at each point  
t = linspace( -1.4, 1.4, 1E3 );  
x = -v.*t;  
  
% Initialize  
thetac = asin( c1./c2 );
```

```
theta0 = 0.99.*[ -thetac, thetac ];
theta1 = 0.*t;

% Find theta1 as function of time
for tCount = 1 : length(t)

    % Current plane position
    D = x(tCount);

    % Function to be solved for theta1 at each point
    thetaVec = linspace( theta0(1), theta0(2), 100 );
    zeroFunction = @(thetaVar) ...
        D - (h.*tan(thetaVar) + d.*tan( asin((c2./c1).*sin(thetaVar)) ) );
    theta1(tCount) = fzero( zeroFunction, theta0 );

end

% Plot theta1 as a function of time
figure();
hold all;

plot( t, 180.*theta1./pi, 'k' );
plot( t, 0.*t + 180.*thetac./pi, '--k', 'LineWidth', 2 );
plot( t, 0.*t - 180.*thetac./pi, '--k', 'LineWidth', 2 );

xlabel( 'Time [s]', 'FontSize', 24 );
ylabel( '$\theta_1$ [deg]', 'FontSize', 24 );

grid on;
box off;

% Compute theta2
theta2 = asin( (c2./c1).*sin(theta1) );

% Compute the instantaneous frequency at the surface
fw = f0.*( 1 - (v./c1).*sin( theta1 ) ).^(-1);

% Plot frequency vs time
figure();
hold all;

plot( t, fw, 'k', 'LineWidth', 3 );
plot( tMeas, fMeas, 'ko', 'MarkerFaceColor', 'k' );

ylabel( 'Frequency [Hz]', 'FontSize', 24 );
xlabel( 'Time [s]', 'FontSize', 24 );
```



```

ylim([60, 80]);
grid on;
box off;

% Compute RMS error
tMin = min(tMeas);
tMax = max(tMeas);

tMeasCompare = tMeas( (tMeas >= tMin) & (tMeas < tMax) );
fMeasCompare = fMeas( (tMeas >= tMin) & (tMeas < tMax) );

fErrors = 0.*tMeasCompare; % Initialize

for tMeasCount = 1 : length( tMeasCompare )

    % Find closest index
    tLoop = tMeasCompare( tMeasCount );
    [~, tInd] = min( abs( t - tLoop ) );

    % Get prediction and estimated frequency at that time.
    fLoop = fMeasCompare( tMeasCount );
    fPredicted = fw( tInd );

    % Store
    fErrors( tMeasCount ) = fPredicted - fLoop;

end

```

## B.2 Problem 2

```

%*****
%
% ASA Student Challenge Problem 2019
%
% Problem 2 – Plots the Doppler shifted frequency as a function of
% the bearing angle from the array for comparison with the provided
% measurements.
%
% Scott Schoen Jr | Georgia Tech | 30 September 2019
%
%*****

clear all
close all
clc

```

```
dataFile = '../media/freqVsBearing.txt';

% Load in data
allData = importdata( dataFile );
fVec = allData( :, 1 );
bearingVec = allData(:, 2 );

% Problem parameters
d = 20; % [m]
h = 151; % [m]
v = 125; % [m/s]
f0 = 68.3; % [Hz]

c1 = 343; % [m/s]
c2 = 1520; % [m/s]

% Define plane position at each point
t = linspace( -1.4, 1.4, 100 );
x = -v.*t;

% Initialize
thetac = asin( c1./c2 );
theta0 = 0.99.*[ -thetac, thetac ];
theta1 = 0.*t;

% Find theta1 as function of time
for tCount = 1 : length(t)

    % Current plane position
    D = x(tCount);

    % Function to be solved for theta1 at each point
    thetaVec = linspace( theta0(1), theta0(2), 100 );
    zeroFunction = @(thetaVar) ...
        D - (h.*tan(thetaVar) + d.*tan( asin( (c2./c1).*sin(thetaVar) ) ) );
    theta1(tCount) = fzero( zeroFunction, theta0 );

end

% Compute theta2 depression angles
theta2 = asin( (c2./c1).*sin(theta1) );
gamma = pi./2 - theta1;
xi = pi./2 - theta2;

% Compute the instantaneous phase
fw = f0.*( 1 - (v./c1).*cos( gamma ) ).^(-1); % At the surface
```

```
% Plot bearing vs frequency
figure();
hold all;

plot( 180.*xi./pi, fw, 'k', 'LineWidth', 3 );
plot( bearingVec, fVec, 'ko', 'MarkerFaceColor', 'k' );

set( gca, 'XTick', 0:30:180 );

ylabel( 'Frequency [Hz]', 'FontSize', 24 );
xlabel( 'Bearing Angle  $\xi$  [deg]', 'FontSize', 24 );

ylim([60, 80]);
grid on;
box off;

lh = legend( '~Prediction [Eq. (2.1)]', '~Measurement', ...
            'FontSize', 18, 'Interpreter', 'LaTeX', 'EdgeColor', 'none' );

% Compute RMS error
xiMin = min(bearingVec);
xiMax = max(bearingVec);

tMin = -0.5;
tMax = 0.5;

xiMeasCompare = bearingVec( (bearingVec >= xiMin) & (bearingVec < xiMax) );
fMeasCompare = fVec( (bearingVec >= xiMin) & (bearingVec < xiMax) );

fErrors = 0.*xiMeasCompare; % Initialize

for xiMeasCount = 1 : length( xiMeasCompare )

    % Find closest index
    xiLoop_deg = xiMeasCompare( xiMeasCount );
    [~, xiInd] = min( abs( 180.*xi./pi - xiLoop_deg ) );

    % Get prediction and estimated frequency at that time.
    fLoop = fMeasCompare( xiMeasCount );
    fPredicted = fw( xiInd );

    % Store
    fErrors( xiMeasCount ) = fPredicted - fLoop;

end
```

```
% Return RMS error
rmsError = rms( fErrors );
```

### B.3 Problem 3

```
%*****
%
% ASA Student Challenge Problem 2019
%
% Script loads the time series hydrophone data and compares the measured
% time frequency data with the calculated shifts.
%
% Scott Schoen Jr | Georgia Tech | 30 September 2019
%
%*****

clear all
close all
clc

% Problem parameters
d = 90; % [m]
h = 150; % [m]
v = 94; % [m/s]
f0 = 68.36; % [Hz]

c1 = 343; % [m/s]
c2 = 1520; % [m/s]

% Load in hydrophone data
dataFile = '../media/hydrophoneSignal.wav';
[hydSig, Fs] = audioread(dataFile);

% Downsample for efficiency
fMax = 30E3; % [Hz]
N = length( hydSig );
fRatio = Fs./fMax;
indsToKeep = round( 1 : fRatio : N );
hydSig = hydSig( indsToKeep );

% Set time of plane overhead
tOffset = 54.6; % [s]
measuredShift = 4.12; % [Hz]

% Normalize
```

```

hydSigNorm = hydSig./max(abs(hydSig));

% Get time signal
N = length( hydSig );
dt = 1./fMax;
tVec = 0 : dt : (N - 1).*dt;

% First sweep through and find the velocity to match this shift
vVec = linspace( 94.5, 96.5, 50 );
deltaf = 0.*vVec;

for vCount = 1 : length( vVec )

    % Define plane position at each point
    t = linspace( -4, 4, 100 );
    x = -vVec(vCount).*t;

    % Initialize
    thetac = asin( c1./c2 );
    theta0 = 0.99.*[ -thetac, thetac ];
    thetal = 0.*t;

    % Find thetal as function of time
    for tCount = 1 : length(t)

        % Current plane position
        D = x(tCount);

        % Function to be solved for thetal at each point
        thetaVec = linspace( theta0(1), theta0(2), 100 );
        zeroFunction = @(thetaVar) ...
            D - (h.*tan(thetaVar) + d.*tan(asin((c2./c1).*sin(thetaVar))));
        thetal(tCount) = fzero( zeroFunction, theta0 );

    end

    % Compute theta2 depression angles
    theta2 = asin( (c2./c1).*sin(thetal) );
    gamma = pi./2 - thetal;
    xi = pi./2 - theta2;

    % Compute the instantaneous phase
    fw = f0.*( 1 - (v./c1).*cos( gamma ) ).^(-1); % At the surface

    % Get frequency range
    deltaf( vCount ) = max(fw) - min(fw);

```

```

end

% Get the optimal velocity
[~, vInd] = min( abs( deltaf./2 - measuredShift ) );

% Compute shifts for that velocity
v = vVec(vInd);
x = -vVec(vInd).*t;

% Initialize
thetac = asin( c1./c2 );
theta0 = 0.99.*[ -thetac, thetac ];
theta1 = 0.*t;

% Find theta1 as function of time
for tCount = 1 : length(t)

    % Current plane position
    D = x(tCount);

    % Function to be solved for theta1 at each point
    thetaVec = linspace( theta0(1), theta0(2), 100 );
    zeroFunction = @(thetaVar) ...
        D - (h.*tan(thetaVar) + d.*tan( asin( (c2./c1).*sin(thetaVar) ) ) );
    theta1(tCount) = fzero( zeroFunction, theta0 );

end

% Compute theta2 depression angles
theta2 = asin( (c2./c1).*sin(theta1) );
gamma = pi./2 - theta1;
xi = pi./2 - theta2;

% Compute the instantaneous phase
fw = f0.*( 1 - (v./c1).*cos( gamma ) ).^(-1); % At the surface

%% Plot spectrogram and overlay the predicted frequency
figure();
hold all;

[s, fVec, tVec] = spectrogram(hydSigNorm, 2.^(15), 2.^(14), [], fMax, 'yaxis');
[T, F] = meshgrid( tVec, fVec );

pcolor( T - tOffset, F, 20.*log10(abs(s)./max(abs(s(:)))) );
colormap( flipud( colormap( gray ) ) );

```

```
shading interp;

plot( t, fw, 'k', 'LineWidth', 5 );

ylim( [55, 85] );
xlim(3.75.*[ -1, 1]);
caxis([-24, 0]);

xlabel( 'Time [s]', 'FontSize', 24 );
ylabel( 'Frequency [Hz]', 'FontSize', 24 );

set( gca, 'XTick', -5:5 );

cbh = colorbar;
cbh.Location = 'North';
cbh.Position = [0.675, 0.8, 0.17, 0.02];
cbh.Ticks = [];

grid on;
box off;
```

## References

- [1] B. G. Ferguson, R. L. Culver, and K. L. Gemba, “International student challenge problem in acoustic signal processing 2019”, *Acoustics Today* **15**, 71–73 (2019).
- [2] R. J. Urick, “Noise signature of an aircraft in level flight over a hydrophone in the sea”, *Journal of the Acoustical Society of America* **52**, 993–999 (1972).
- [3] B. Ferguson and G. Speechley, “Acoustic detection and localization of an ASW aircraft by a submarine”, *The United States Navy Journal of Underwater Acoustics* **39**, 25–41 (1989).
- [4] A. Kwan, J. Dudley, and E. Lantz, “Who really discovered Snell’s law?”, *Physics World* **15**, 64 (2002).
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- [6] A. D. Pierce, *Acoustics: an Introduction to Its Physical Principles and Applications* (Acoustical Society of America, Melville, NY) (1989).